# Introduction

This project is created as a part of IE 515 – Graphs and Network Flows class of Boğaziçi University Industrial Engineering department. The aim of the project is implementing network flow algorithms and compare their performances. To achieve that aim, a minimum cost flow problem is modeled and solved by using two basic minimum cost flow algorithms namely cycle-cancelling and successive shortest path algortihms. The model used as an example problem is building evacuation model which is a dynamic flow application.

In dynamic flows, the underlying solution idea is basically modeling the network as a static network, and creating its time expanded replicas. In building evacuation case, a building is represented as a static network. The nodes represent locations of buildings such as offices, work centers, hallways etc. The arcs represent the passages between those locations. Arcs have capacity and cost attributes. Capacities and costs respectively represent the number of people that can pass per unit time, and time required for passing that arc.

Time expanded replica is created by simply creating a copy of the static network for each time period. Then, in the replicated network, each node will have a copy of number of time periods which are represented by (i,p). In the replica, arcs are connected to the nodes according to their costs. For example, let the cost of arc (i,j) in the static network be t. Then, for every (i,p) node in the replicated network, there will be an arc ((i,p),(j,p+t)).

The aim of the model is finding the minimum time required for the building to be evacuated. We can find that objective by solving a minimum cost flow problem in the replicated network. We can model the minimum cost flow problem by adding super-source and super-sink nodes. Super-source will be connected to each replica of source with uncapacitated arcs of zero cost. Additionally, super-sink will be connected nodes with uncapacitated arcs of increasing cost. Supply of super-source and demand of super-sink will represent the number of people to be evacuated. In the solution of the minimum cost flow problem on the modified replicated model, the sink node with the maximum time period connected to super-sink will give us the minimum time needed to evacuate the building.

# Approach

As it is mentioned in the introduction, two minimum cost flow algorithms will be implemented to solve the building evacuation model. The first algorithm implemented in cycle-cancelling algorithm. Cycle-cancelling algorithm firstly establishes a feasible flow in the network. To find the feasible flow, a max-flow problem on the network is solved. In the max flow problem, a source node connected to supply nodes with arcs of capacity equal to the supply and cost zero. Also, a sink node is connected from each demand node with arc capacities equal to their demand with cost of zero. Then, by solving a max flow problem on the modified network, the feasible flow is found. To find the feasible flow, max-flow labeling algorithm is implemented. After establishing feasible flow, negative cycles are found and flows augmented on negative cycles until no negative cycles left. To find negative cycles, Bellman-Ford algorithm is implemented.

The second algorithm implemented is successive shortest path algorithm. In this algorithm, the idea is finding shortest paths on the network with respect to reduced arc costs, and augmenting flow on the shortest paths until all demand is satisfied. The algorithm solves a shortest path problem in each iteration, and shortest paths are found by implementing Bellman-Ford algorithm.

# Analysis of the Algorithms

We know from the negative cycle optimality conditions that if there is no negative cycle, then the network is optimal. Cycle-cancelling algorithm strictly finds a negative cost cycle and augments flow on the cycle until no negative cycles left. Therefore, the algorithm converges to a optimal solution. The number of iterations required for the algorithm in the worst case is 2mCU where m, C and U are number of arcs, max cost, and max capacity in the network. At each iteration, we find negative cycles by solving a shortest path problem by using Bellman-Ford algorithm, which is of complexity O(nm). As a result, overall complexity of the algorithm is O(nm2CU) which is a pseudopolynamial complexity.

The successive shortest path algorithm sends flow from excess nodes to deficit nodes at each iteration. Therefore, maximum number of iterations required is nB where n is the number of nodes and B is the maximum supply in the network. At each iteration we solve a shortest path problem. Thus, the complexity of the algorithm is O(nB\*S(n,m,C)) where S(n,m,C) represents the complexity of the shortest path algorithm used. In this project, the algorithm used is Bellman-Ford, therefore, successive shortest path algorithm has the complexity of O(n2mB). From the theoretical analysis, we expect successive shortest path algorithm to be faster than cycle-cancelling algorithm.

# Experiment Results & Discussion

# Future Work & Recommendations

In the project, a simple idea of building evacuation models is used as an example. Synthetic networks are generated and solved to compare algorithms. However, there are more realistic models exist. As an example, Chalmet et al[[1]](#footnote-1) represents more realistic approach to building evacuation models. By using more complex models and real-life examples of buildings, the work may create a practical value.

In the project, the algorithms used for solving problems are mostly the basic algorithms. We can increase the performance of algorithms by using more efficient ideas. For example, we can increase the performance successive shortest path algorithm by using Dijkstra’s algorithm instead of Bellman-Ford. This may increase the speed significantly. Also, we can increase the performance by using more efficient data structures as well.

The implemented algorithms finds optimal solutions for feasible problems. By adding new rules, we may detect infeasibility and unboundedness, therefore, increase the capacilities of algorithms. Lastly, data entry and and output analysis processess can be designed in a more user-friendly manner. As a result, the implemented algorithms can be more efficiently used for solution of different problems.

# Conclusion

In this project, building evacution models are analyzed and solved by using minimum cost flow algorithms. There are several significant outcomes of this task. Firstly, the concept of dynamic flows and the methods for approaching those problems are introduced. Secondly, the usage of minimum cost flow problem for modeling the dynamic flows is experienced. Thirdly, the minimum cost flow algorithms are implemented and implementation experience is gained. Lastly, the process of approaching a problem, modeling it by using the theoretical background learned throughout the semester, and solving it by implementing algorithms is experienced. I believe that experience is very valuable.

# References

1. Chalmet, L. G., R. L. Francis, and P. B. Saunders. "Network models for building evacuation." Management science 28.1 (1982): 86-105.
2. Ahuja, Ravindra K., Tomas L. Magnanti, and James B. Orlin. "Network flows. 1993."

1. Chalmet, L. G., R. L. Francis, and P. B. Saunders. "Network models for building evacuation." Management science 28.1 (1982): 86-105. [↑](#footnote-ref-1)